SPH4U **Relativistic Momentum & Matter-Energy Equivalence**

Recall that the theory of special relativity says that weird things happen at high speeds:
- time slows down
- length contracts

Are there any other variables that change? Consider momentum and mass...

Relativistic Momentum

– subatomic particles are zipping about at relativistic speeds ($v > 0.7c$)
– momentum is given by

$$p' = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

– m_0 is the **rest mass** (the mass measured by the observer at rest)

Ex// Linear accelerators accelerate charged particles to nearly the speed of light. A proton is accelerated to 0.999 994c.

(a) Determine the magnitude of the relativistic momentum.

(b) Compare the relativistic and classical momentum of the proton.

Note:

$$\begin{aligned} p' &= \gamma p_0 \\ (mv)' &= \gamma (mv)_0 \qquad but \quad v' = v_0 \\ m' &= \gamma m_0 \end{aligned}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- as $v \to c$, the mass becomes infinitely heavy
- infinitely heavy objects require infinite amount of energy to move them
- nothing can move faster than c
- also, mass can't be smaller than 1 unit
- problem: conservation of energy?

Matter-Energy Equivalence

To keep a conservation of energy law, we must change the definition of energy. Why?

Consider classical kinetic energy $E_k = ½ mv^2$. We assumed that mass didn't change, but at relativistic speeds it does. Classically, we can have limitless E_k. Mass and energy are separate ideas and are conserved separately. What happens to the energy at relativistic speeds?

Well, *c* is the upper limit. As you go faster and faster, where does the mass come from?

Einstein: "Matter is a localized clump of energy."

$E_{tot} = E_k + E_{rest} = \gamma m_o c^2$ $E_k = (\gamma - 1) m_o c^2$ $E_{rest} = m_o c^2$

We no longer have conservation of mass or conservation of energy, but conservation of (matter+energy), which ALWAYS applies.

New unit of energy: electron-volt (eV). 1 eV = 1.60×10^{-19} J

Ex// If the 0.50 kg mass of a ball at rest were totally converted to another form of energy, what would the energy output be?

Ex// An electron moves at 0.860*c* in a lab. Calculate the electron's rest energy, total energy, and kinetic energy in the lab frame, in eV.